$$= \left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)^{\frac{1}{3}}$$

$$= \left(\frac{\sqrt{3} - i}{2}\right)^{\frac{1}{3}}$$

Dot and Cross Product

أولا: عملية dot ويرمز لها بالرمز O وتعرف

zoz يمثل الجزء الحقيقي

$$z_1 o z_2 = |z_1||z_2| \cos\theta$$
$$= x_1x_2 + y_1y_2$$
$$= Re(\bar{z}_1 z_2)$$
$$= \frac{1}{2}(\bar{z}_1 z_2 + z_1 \bar{z}_2)$$

ثانيا: عملية cross ويرمز لها بالرمز x وتعرف

$z \times z$ يمثل الجزء التخيلي

$$z_1 \times z_2 = |z_1||z_2| \sin\theta$$
$$= x_1y_2 - y_1x_2$$
$$= I_m(\bar{z}_1 z_2)$$
$$= \frac{1}{2i}(\bar{z}_1 z_2 - z_1 \bar{z}_2)$$

Such that

$$\bar{z}_1 z_2 = Re(\bar{z}_1 z_2) + iI_m(\bar{z}_1 z_2)$$
$$= (z_1 o z_2) + i(z_1 \times z_2)$$
$$= |z_1||z_2| \cos\theta + i|z_1||z_2| \sin\theta$$
$$= |z_1||z_2|(\cos\theta + i\sin\theta)$$
$$= |z_1||z_2|e^{i\theta}$$

EX2:

جد الجذور التكعيبية للعدد $\sqrt{6}-i\sqrt{2}$

Sol:

$$r=\sqrt{6+2}=2\sqrt{2}=2^{3/2}$$

$$\cos\theta=\frac{\sqrt{3}}{2}\ ,\qquad \sin\theta=\frac{-1}{2}\quad\Rightarrow\theta=\frac{-\pi}{6}=-30$$

$$z=\sqrt[3]{2^{\frac{3}{2}}}\left(\cos\frac{\frac{-\pi}{6}+2k\pi}{3}+i\sin\frac{\frac{-\pi}{6}+2k\pi}{3}\right)$$

$$z=\sqrt{2}\left(\cos\frac{-\pi}{18}+\frac{2k\pi}{3}+i\sin\frac{-\pi}{18}+\frac{2k\pi}{3}\right)$$

$$z_0=\sqrt{2}\left(\cos\frac{-\pi}{18}+\frac{2k\pi}{3}+i\sin\frac{-\pi}{18}+\frac{2k\pi}{3}\right)$$

$$z_1=\sqrt{2}\left(\cos\frac{-\pi}{18}+\frac{2\pi}{3}+i\sin\frac{11\pi}{18}\right)$$

$$z_2=\sqrt{2}\left(\cos\frac{11\pi}{18}+\frac{2\pi}{3}+i\sin\frac{23\pi}{18}\right)$$

EX3:

$$\left[\frac{\mp1\mp i\sqrt{3}}{2}\right]^6=1$$

If $z=\frac{1-i\sqrt{3}}{2}\Rightarrow r=\sqrt{\frac{1}{2}+\frac{3}{4}}=1$

$$\cos\theta=\frac{1}{2}\ ,\qquad \sin\theta=\frac{-\sqrt{3}}{2}\ \Rightarrow\theta=\frac{-\pi}{3}+2k\pi\quad ,\ k=0,1,2,\dots$$

$$z=\left(\cos\frac{-\pi}{3}+2k\pi+i\sin\frac{-\pi}{3}+2k\pi\right)^6$$

$$z=\left(\cos\frac{\pi}{18}+2k\pi-i\sin\frac{\pi}{18}+2k\pi\right)$$

$$=\cos 10-i\sin 10$$

$$=\left(\cos\frac{30}{3}-i\sin\frac{30}{3}\right)^{\frac{1}{3}}$$

ويمكن إيجاد جذور المعادلة كما في الأمثلة التالية :

EX1:

جد جذور المعادلة $z^5 = -32$

Sol:

$$z^5 = -32 = (-2)^5$$

$$z = -2 = -2 + 0i$$

$$|z| = r = \sqrt{x^2 + y^2} = 2$$

$$\cos\theta = \frac{-2}{2} = -1 \quad , \quad \sin\theta = 0 \quad , \quad \theta = \pi$$

$$\therefore z = 2(\cos(\pi + 2k\pi) + i\sin(\pi + 2k\pi))$$

حسب طريقة ديموفر

$$z^5 = 2^5(\cos\theta + i\sin\theta)^5$$

$$= 2^5(\cos 5\theta + \sin 5\theta)$$

$$5\theta = \pi + 2k\pi$$

نقسم على 5 وعدد الدورات يبقى ثابت

$$\theta = \frac{\pi}{5} + 2k\pi = \frac{\pi + 2k\pi}{5}$$

$$z = 2\left(\cos\frac{\pi}{5} + 2k\pi + i\sin\frac{\pi}{5} + 2k\pi\right)$$

$$k = 0 \Rightarrow z_1 = 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$$

$$k = 1 \Rightarrow z_2 = 2\left(\cos\frac{\pi + 2\pi}{5} + i\sin\frac{\pi + 2\pi}{5}\right) = 2\left(\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right)$$

$$k = 2 \Rightarrow z_3 = 2(\cos\pi + i\sin\pi)$$

$$k = 3 \Rightarrow z_4 = 2\left(\cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}\right)$$

$$k = 4 \Rightarrow z_5 = 2\left(\cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}\right)$$