Example:

Given that : $f(x) = \frac{x^3}{1-x^4}$, find f '(x)

Solution:

$$f'(x) = \frac{(1-x^4)\frac{d}{dx}(x^3) - (x^3)\frac{d}{dx}(1-x^4)}{(1-x^4)^2}$$

$$= \frac{(1-x^4)(3x^2) - (x^3)(-4x^3)}{(1-x^4)^2}$$

$$= \frac{3x^2 - 3x^6 + 4x^6}{(1-x^4)^2}$$

$$= \frac{3x^2 + x^6}{(1-x^4)^2}$$

6- The Chain Rule

F'(x) = f'(g(x)) · g'(x)

Example :

Suppose we wish to differentiate y = (2x − 5)10

F'(x) = f'(g(x)) · g'(x)

$F'(x) = 10(2x-5)^9 . 2$

$=20(2x -5)^9$

Example:

Find the derivative of the function $g(t) = (t -2 /2t+1)^9$

Combining the Power Rule, Chain Rule, and Quotient Rule, we get

$g'(t) = 9 (t -2 /2t+1)^8 \, d/ dt \, (t-2/ 2t+1)$

$= 9(-2/ 2t+1)^8 (2t+1 \cdot 1-2(t-2) /(2t+1)^2 \quad = 45(t-2)^8/ (2t+1)^{10}$

7- Differentiation of Trigonometric Functions

Derivation of sin x: (sin x)' = cos x
Derivative of cos x: (cos x)' = -sin x
Derivative of tan x: (tan x)' = $\sec^2$ x
Derivative of cot x: (cot x)' = -cosec^2 x
Derivative of sec x: (sec x)' = sec x.tan x
Derivative of cosec x: (cosec x)' = -cosec x.cot x

Example :

$$y = 3\sin x - 4\cos x$$

$$D\{cf(x)\} = cf'(x)$$

$$y' = 3\ D\{\sin x\} - 4\ D\{\cos x\}$$

$$= 3(\cos x) - 4(-\sin x)$$

$$= 3\cos x + 4\sin x$$

Example:

What is d/dx sin(x^2) ?

The Chain Rule says:

the derivative of f(g(x)) = f'(g(x))g'(x)

The individual derivatives are:

- f'(g) = cos(g)
- g'(x) = 2x

So:

d /dx sin(x^2) = cos(g(x)) (2x)

= 2x cos(x^2)

Example :

Find the derivative of cos(x)sin(x)

The Product Rule says:

the derivative of fg = f g’ + f’ g

In our case:

- f = cos
- g = sin

We know (from the table above):

- d/dx cos(x) = −sin(x)
- d/dx sin(x) = cos(x)

the derivative of cos(x)sin(x)

= cos(x)cos(x) − sin(x)sin(x)

$= \cos^2(x) - \sin^2(x)$

Example :

What is the derivative of cos(x)/x ?

In our case:

f = cos

g = x

We know (from the table above):

f' = −sin(x)

g' = 1

So:

the derivative of cos(x) / x = $= x(-\sin(x)) - \cos(x)(1) / x^2$

$= -x\sin(x) + \cos(x) / x^2$

Example :

What is $d / dx \sin(x^2)$?

The Chain Rule says:

the derivative of $f(g(x)) = f'(g(x))g'(x)$

The individual derivatives are:

$f'(g) = \cos(g)$

$g'(x) = 2x$

So: $d / dx \sin(x^2) = \cos(g(x))\,(2x)$

$= 2x\cos(x^2)$

Example :

Differentiate $y = \csc x \cot x$. Apply the product rule.

Then

$$y' = \csc x \; D\{\cot x\} + D\{\csc x\} \; \cot x$$

$$= \csc x(-\csc^2 x) + (-\csc x \cot x)\cot x$$

$$= -\csc^3 x - \csc x \cot^2 x$$

$$= -\csc x(\csc^2 x + \cot^2 x)$$

Example :

Differentiate $y = \frac{\sin^2 x}{\cos^2 x}$. To avoid using the chain rule, first rewrite the problem as

$$y = \frac{\sin^2 x}{\cos^2 x}$$

$$= \frac{(\sin x)^2}{(\cos x)^2}$$

$$= \left(\frac{\sin x}{\cos x}\right)^2$$

$$= (\tan x)^2$$

$$= \tan x \tan x \,.$$

Now apply the product rule. Then

$$y' = \tan x \; D\{\tan x\} + D\{\tan x\} \;\; \tan x$$

$$= \tan x(\sec^2 x) + (\sec^2 x)\tan x$$

$$= 2\sec^2 x \tan x$$

Example :

Differentiate $f(x) = \cos(2x) + \sin^2 x$. Apply the chain rule to both functions. (If necessary, review the section on the chain rule .) Then

$$f'(x) = -\sin(2x)\; D\{2x\} + 2\sin x \; D\{\sin x\}$$

$$= -\sin(2x)(2) + 2\sin x(\cos x)$$

$$= -2\sin(2x) + (2\sin x \cos x)$$