Chapter Three

1-The Exponential Function

Exponent Rules

Exponent rules are those laws that are used for simplifying expressions with exponents. Many arithmetic operations like addition, subtraction, multiplication, and division can be conveniently performed in quick steps using the laws of exponents. These rules also help in simplifying numbers with complex powers involving fractions, decimals, and roots.Let us learn more about the different rules of exponents, involving different kinds of numbers for the base and exponents.

3-1-Exponent Rules Chart

The rules of exponents explained above can be summarized in a chart as shown below.

Law	Example
$x^1 = x$	$6^1 = 6$
$x^0 = 1$	$7^0 = 1$
$x^{-1} = 1/x$	$4^{-1} = 1/4$
$x^m x^n = x^{m+n}$	$x^2 x^3 = x^{2+3} = x^5$
$x^m/x^n = x^{m-n}$	$x^6/x^2 = x^{6-2} = x^4$
$(x^m)^n = x^{mn}$	$(x^2)^3 = x^{2\times3} = x^6$
$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
$(x/y)^n = x^n/y^n$	$(x/y)^2 = x^2 / y^2$
$x^{-n} = 1/x^n$	$x^{-3} = 1/x^3$

And the law about Fractional Exponents:

$x^{m/n} = n\sqrt{x^m}$	$x^{2/3} = 3\sqrt{x^2}$
$= (n\sqrt{x}\ )^m$	$= (3\sqrt{x}\ )^2$

Example 1:

Simplify the exponential function $5^x - 5^{x+3}$.

Solution:

Given exponential function: $5^x - 5^{x+3}$

From the properties of an exponential function,

we have $a^x \times a^y = a^{(x+y)}$

So, $5^{x+3} = 5^x \times 5^3 = 125\times5^x$

Now, the given function can be written as

$$5^x - 5^{x+3} = 5^x - 125\times5^x$$

$$= 5^x(1 - 125)$$

$$=5^x(-124)$$

$$= -124(5^x)$$

Example 2:

Find the value of x in the given expression: $4^3\times (4)^{x+5} = (4)^{2x+12}$.

Solution:

Given,

$$4^3\times (4)^{x+5} = (4)^{2x+12}$$

From the properties of an exponential function, we have $a^x \times a^y = a^{(x+y)}$

$$\Rightarrow (4)^{3+x+5} = (4)^{2x+12}$$

$$\Rightarrow (4)^{x+8} = (4)^{2x+12}$$

Now, as the bases are equal, equate the powers.

$$\Rightarrow x+8 = 2x+12$$

$$\Rightarrow x - 2x = 12 - 8$$

$$\Rightarrow - x = 4$$

$\Rightarrow x = -4$ Hence, the value of x is –4.

Example 3:

Simplify: $(3/4)^{-6} \times (3/4)^{8}$.

Solution:

Given: $(3/4)^{-6} \times (3/4)^{8}$

From the properties of an exponential function, we have $a^{x} \times a^{y} = a^{(x+y)}$

Thus, $(3/4)^{-6} \times (3/4)^{8} = (3/4)^{(-6+8)}$

$= (3/4)^{2}$

$= 3/4 \times 3/4 = 9/16$

Hence, $(3/4)^{-6} \times (3/4)^{8} = 9/16$.

Example 4:

Solve the exponential equation: $(1/6)^{x-5} = 216$.

Solution:

Given exponential equation is:

$(1/6)^{x-5} = 216$

$\Rightarrow (1/6)^{x-5} = 6^{3}$

From exponent formulas, we have $a^{-x} = 1/a^{x}$

So, $(1/6)^{x-5} = 6^{-(x-5)}$

$6^{-(x-5)} = 6^{3}$

Now, as the bases are equal, equate the powers.

$\Rightarrow -(x-5) = 3$

$\Rightarrow -x + 5 = 3$

$\Rightarrow -x = 3 - 5 = -2$

$\Rightarrow x = 2$

Hence, the value of x is 2.

2- The logarithmic function

1) $\ln xy = \ln x + \ln y$	1) $\log a\; xy = \log a\; x + \log a\; y$
2) $\ln x\; y = \ln x - \ln y$	2) $\log a\; x\; y = \log a\; x - \log a\; y$
3) $\ln x\; y = y \cdot \ln x$	3) $\log a\; x\; y = y \cdot \log a\; x$
4) $\ln e\; x = x$	4) $\log a\; a\; x = x$
5) $e \ln x = x$	5) $a \log a\; x = x$
6) $\ln e = 1$	6) $\log a\; a = 1,\; for\ all\ a > 0$
7) $\ln 1 = 0$	7) $\log a\; 1 = 0,\; for\ all\ a > 0$

xponential function	Logarithmic function	Read as
$8^2 = 64$	$\log_8 64 = 2$	log base 8 of 64
$10^3 = 1000$	$\log 1000 = 3$	log base 10 of 1000
$10^0 = 1$	$\log 1 = 0$	log base 10 of 1
$25^2 = 625$	$\log_{25} 625 = 2$	log base 25 of 625
$12^2 = 144$	$\log_{12} 144 = 2$	log base 12 of 144

Example 1

Rewrite exponential function $7^2 = 49$ to its equivalent logarithmic function.

Solution:

Given $7^2 = 49$.

Here, the base = 7, exponent = 2 and the argument = 49. Therefore, $7^2 = 64$ in logarithmic function is; $\Rightarrow \log_7 49 = 2$