Example: Find the value of $\sin 75°$.

Solution:

The aim is to find the value of $\sin 75°$.

Her we can use the formula $\sin(A + B) = \sin A.\cos B + \cos A.\sin B$.

Here we have $A = 30°$ and $B = 45°$

$\sin 75° = \sin(30° + 45°)$

$= \sin 30°.\cos 45° + \cos 30°.\sin 45°$

$= (1/2)\ (1/\sqrt{2}) + (\sqrt{3}/2)\ (1/\sqrt{2})$

$= 1/2\sqrt{2} + \sqrt{3}/2\sqrt{2}$

$= (\sqrt{3} + 1) / 2\sqrt{2}$

2-7-Half-Angle Identities

$\sin A/2 = \pm\sqrt{[(1 - \cos A) / 2]}$

$\cos A/2 = \pm\sqrt{[(1 + \cos A) / 2]}$

$\tan A/2 = \pm\sqrt{[(1 - \cos A) / (1 + \cos A)]}$ (or) $\sin A / (1 + \cos A)$ (or) $(1 - \cos A) / \sin A$

Example :

Use an appropriate half angle formula to find the exact value of :

$\cos \pi/8$.

Solution:

Using the half angle formula of cos,

$\cos A/2 = \pm\sqrt{[(1 + \cos A)/2]}$

We know that $\pi/8 = 22.5°$.

Substitute A = 45° on both sides,

$\cos 45°/2 = \pm\sqrt{[(1 + \cos 45°)/2]}$

From Trig chart, we know that $\cos 45° = \sqrt{2}/2$.

$\cos 22.5° = \pm\sqrt{[1 + (\sqrt{2}/2)/2]}$

$\cos 22.5° = \pm\sqrt{[(2 + \sqrt{2})/(2 \times 2)]}$

$\cos 22.5° = \pm\sqrt{(2 + \sqrt{2})}/2$

But 22.5° lies in quadrant I and hence cos 22.5° is positive. Thus,

$\cos 22.5° = \sqrt{(2 + \sqrt{2})}/2$

2-8-Double Angle Identities

$\sin(2x) = 2\sin(x)\cos(x) = [2\tan x/(1+\tan^2 x)]$
$\cos(2x) = \cos^2(x)-\sin^2(x) = [(1-\tan^2 x)/(1+\tan^2 x)]$
$\cos(2x) = 2\cos^2(x)-1 = 1-2\sin^2(x)$
$\tan(2x) = [2\tan(x)]/[1-\tan^2(x)]$
$\cot(2x) = [\cot^2(x) - 1]/[2\cot(x)]$
$\sec(2x) = \sec^2 x/(2-\sec^2 x)$
$\mathrm{cosec}(2x) = (\sec x.\ \mathrm{cosec}\ x)/2$

Example 6:

If $\tan A = 3/4$, find the values of $\sin 2A$, $\cos 2A$, and $\tan 2A$.

Solution:

Since the value of tan A is given, we use the double angle formulas for finding each of sin 2A, cos 2A, and tan 2A in terms of tan.

$$\sin 2A = 2\tan A / 1+\tan^2 A = 2(3/4) / 1+(3/4)^2 = 24/25$$

$$\cos 2A = 1-\tan^2 A / 1+\tan^2 A = 1-(3/4)^2 / 1+(3/4)^2 = 7/25$$

$$\tan 2A = 2\tan A / 1-\tan^2 A = 2(3/4) / 1-(3/4)^2 = 24/7$$

Answer:

$\sin 2A = 24/25$, $\cos 2A = 7/25$, and $\tan 2A = 24/7$

2-9-Triple Angle Identities

$\sin 3x = 3\sin x - 4\sin^3 x$
$\cos 3x = 4\cos^3 x - 3\cos x$
$\tan 3x = [3\tan x - \tan^3 x]/[1-3\tan^2 x]$

Example:-

$\sin 3x = \sin(2x+x)$

$= \sin(2x)\cos x + \cos(2x)\sin x$

$= (2\sin x\cos x)\cos x + (\cos^2 x - \sin^2 x)\sin x$

$= 2\sin x\cos^2 x + \cos^2 x\sin x - \sin^3 x$

$= 3\sin x\cos^2 x - \sin^3 x = 3\sin x(1-\sin^2 x) - \sin^3 x$

$= 3\sin x - 4\sin^3 x$

2-10-Product identities

2sinx·cosy=sin(x+y)+sin(x−y)

2cosx·cosy=cos(x+y)+cos(x−y)

2sinx·siny=cos(x−y)−cos(x+y)

sinx+siny=2sin((x+y)/2) . cos((x−y)/2)

sinx−siny=2cos((x+y)/2) . sin((x−y)/2)

cosx+cosy=2cos((x+y)/2) . cos((x−y)/2)

cosx−cosy=−2sin((x+y)/2 . sin((x−y)/2)

Example:

Use sum to product formula to express cos 8x + cos 2x as the product.

Solution:

To express cos 8x + cos 2x as the product, we will use the formula

cos A + cos B = 2 cos [(A + B)/2] cos [(A - B)/2].

Substituting A = 8x and B = 2x into the formula, we have

cos 8x + cos 2x = 2 cos [(8x + 2x)/2] cos [(8x - 2x)/2]

= 2 cos (10x/2) cos 6x/2

= 2 cos 5x cos 3x

Answer:

Hence,

cos 8x + cos 2x = = 2 cos 5x cos 3x using the sum to product formula.