Chapter three

Ideals

:Definition :3-1

Anon-empty subset I of a ring (R , + , .) is called an ideal of R

a , b ∈ I implies a – b ∈ I ∀ ①

.∀ a ∈ I and r ∈ R imply a . r ∈ I and r . a ∈ I ②

:Definition :3-1

Anon-empty subset I of a ring (R , + , .) is called a right (left) ideal if

. ∀ a , b ∈ I implies a – b ∈ I ①

. ∀ a ∈ I and r ∈ R imply a . r ∈ I (r . a ∈ I) ②

; Clearly

a right ideal or left ideal is a subring of R and . every ideal is both right and left, so an ideal

.is sometimes called a two-sided ideal

Trivially, in a commutative ring every right ideal or , left ideal is two-sided. In every ring R and R are ideals called trivial ideals (0)

:Examples:3-3 .

is ideal in a ring ((3) , +12 , (12· , 12+ , (3))❶
.• 12) So it's a subring

In general (a) = { na : n ∈ Z } . ((a) ,+ , .) is an ❷
. ideal. in aring (Z, + , .)

Because na - ma = (n - m) a ∈ (a) and in (ma) =
. (mn) a ∈ (a) where n , m ∈ Z

for example (2) = Ze i.e. ((2) ,+, .) or (Ze ,
. + , .) is an ideal of (Z, + , .)

let (R, + , .) be a ring and I = { f ∈ R : f (I) = 0 } (f ❸
. - g)(I) = f(I) - g(I) = 0 – 0 = 0 ∀ h ∈ R , f, g ∈ I

. h(I) . f(I) = h(I) . 0 = 0 = (I)(h . f)

. Thus (I , + , .) is an ideal of a ring (R, + , .)

Definition: A ring (R, + , .) is called a simple :3-4
. ring if it has no proper ideal of it

.(or it has only the triveal ideals)