Examples 1-6

is a ring , such that R 2 = R x R { (a , (· ,+ , 2)
b) : a , b ∈ R }

Solution : Let (a , b) , (c , d) , (h , L) ∈ R^2

R^2 closed ∋ (a + c , b + d) = (c, d) + (a, b) ❶

h,) + (a + c , b + d) = (h , L) +[(c , d) + (a , b)]❷
a + (c + h) , b + (d) = (h , (b + d) + L + (a + c)) = (L
(+ L)

h ,) + (c , d)) + (a , b) = (e + h , d + L) + (a , b) =
associa (L

(c + a , d + b) = (a + c , b + d) = (c , d) + (a , b)❸
commutative (a , b) + (c , d) =

∀ (a , b) E R^2 ∈ (0 , 0) ∈ R^2 ∋ (a , b) + (0, 0) = ❹
(a + 0 , b + 0) = (a , b) and (0 , 0) + (a , b) = (0 + a
, 0 + b) = (a , b) identity

∀ (a , b) ∈ R^2 ∃ (-a , -b) ∈ R^2 ∋ (a , b) + (-a ,-b) ❺
= (a + (-a) , b + (-b))

and (-a , -b) + (a , b) = (0 , 0) . inverse (0 , 0) =

R^2 closed ∋ (a . c , b . d) = (c , d) · (a , b)❻
under multiplication 8

h) . (a . c , b . d) = (h , L) . ((c , d) . (a . b))❼ ,(h , (b . d) . L . (a . c)) = (, L

(c . h , d . L) . (a , b) = (a . (c . h) , b . (d . L)) = associa ((h , L) . (c , d)) . (a . b) =

. Thus (R 2 , +, ∙) is a ring

(c . a , d . b) = (a . c , b . d) = (c , d) . (a , b)❽ commutative (a , b) . (c , d) =

∀ (a , b) ∈ R^2 ∃ (1 , 1) ∈ R^2 ∋ (a , b) . (1, 1) = (❾ a . 1 , b . 1) = (a , b) . So (1 , 1) . (a , b) = (a , b) identity

Therefore (R 2 , +, ∙) is a commutative ring with . identity