:Examples (7- 4)

, is a belian ring with identity (Z , +n , n)

where n is a posative integer solution: Let [a] , [b] , [c] ∈ Zn

.n [b] = [a + b] ∈ Zn (closed)+ [a]❶

n [c] = [a + b] +n [c] = [(a + b) + c+ (n [b]+ [a])❷] = [a + (b + c)] = [a] +n [b + c] = [a] +n ([b] +n [c]) (associative)

n [b] = [a + b] = [b + a] = [b] +n [a] + [a]❸ (commutative)

n [0] = [a + 0] = [a] , [0] +n [a] = [0+ a] = [a] + [a]❹ ∴ ∀ [a] ∈ Zn ∃ [0] ∈ Zn ∋ [a] +n [0] = [0] +n [a] = [a] (identity)

n [-a] = [a + (-a)] = [0] , [-a] +n [a] = [-a + + [a]❺ a] = [0] ∀ [a] ∈ Zn ∃ [-a] ∈ Zn ∋ [a] +n [-a] = [-a] + [a] = [0] (inverse)

. Thus (n , +n) is a belian group

n [b] = [a • b] ∈ Zn (closed under) 9· [a]❻

n [c] = [a .b] •n [c] = [(a • b) . c] · (n [b]· [a])❼ = [a . (b . c)] =[a] •n [b . c] = [a] •n ([b] •n [c]) .(associative)

n [b] = [a • b] = [b • a] = [b] •n [a] · [a]❽
(commutative)

n [1] = [a. 1] = [a] , [1] •n [a] = [1 • a] = [a]· [a]❾

∀ [a] ∈ Zn ∃ [1] Zn ∋ [a] •n [1] = [1] •n [a] = ∴ [a] (identity)

Therefore (Z , +n , n) is a commutative ring with identity

Definitions: A ring in the form ({0} , + , •) is 1-8 called Zero ring

: Ramark :9 -1

.Let (R , + , •) be a ring, and let a , b ∈ R

Then -(a. b) = a . (-b) = (-a). b ①

a . b = (b-) . (a-)②

a . (b - c) = a . b – a . c and (b - c) . a = b . a – c . ③ a

Proof: ① a .b + a .(-b) = a (b + (-b)) = a . 0 = 0 ⇒ -(a . b) = a . (-b) a . b + (-a) . b = (a + (-a)) . b = 0 . b = 0 b Hence $-(a \cdot b) = a \cdot (-b) = (-a) \cdot b \cdot (a-) = (a \cdot b)-$ $\Rightarrow$

$a \cdot b$ 10 $= ((a \cdot b)-)- = (b \cdot (a-))- = (b-) \cdot (a-)$②

$a \cdot b = (b-) \cdot (a-)$ $\Rightarrow$

$a \cdot (b - c) = a \cdot (b + (-c)) = a \cdot b + a \cdot (-c) = a \cdot b +$ ③
$(-(a \cdot c)) = a \cdot b - a \cdot c$

and $(b - c) \cdot a = (b + (-c)) \cdot a = b \cdot a + (-c) \cdot a = b \cdot a +$
$(-(c \cdot a)) = b \cdot a - c \cdot a$