

:Definitions 1-3

A ring $(R, *, \circ)$ is called a belian ring if (R, \circ) is a belian semi-group

A ring (R, \circ) is called a ring with identity if (2) there exists an identity element $E \in R$, such that $a \circ e = e \circ a = a, \forall a \in R$

A ring of invertible elements is a ring $(R, *, \circ)$ (3) if $(R, *)$ is a group of invertible elements

I.e. each element of (R, \circ) has a unique inverse. And is denoted by $(R^*, *, \circ)$

Examples $(R^*, +, \cdot)$ is a ring of invertible element $(R, +, \cdot), (Z, +, \cdot), (Ze, +, \cdot), (Q, +, \cdot)$

.Theorem : Each ring has a unique identity

Proof: Suppose $(R, *, \circ)$ is a ring has identity e and e'

Hence $\forall a \in R$, we get $a \circ e = a$ and $a \circ e' = a \therefore a \circ e = a \circ e'$

Thus R has a unique identity

Theorem : Each non-zero element in the ring has a unique inverse

. Proof: Let $(R, *, \circ)$ be a ring and let $0 \neq a \in R$

: Suppose $a^{-1}, \alpha \in R$ are inverse of a implies

$$a \circ \alpha = e \text{ and } a^{-1} \circ a = e \text{ So } a \circ a^{-1} = a \circ \alpha \circ a^{-1} \circ (a \circ a^{-1}) = a^{-1} \circ (a \circ \alpha) \circ (a^{-1} \circ a) \circ a^{-1} = (a^{-1} \circ a) \circ \alpha$$

$$e \circ a^{-1} = e \circ \alpha \circ a^{-1} = \alpha \implies$$

.Thus a has a unique inverse