

## Chapter six

### Numerical Integration

Example: find  $\left( \int_1^6 (2 + \sin(2\sqrt{x})) dx \right)$  by using Trapezoidal formula and evaluate the error of the method where  $n=10$ .

Solution:  $a=1, b=6$  ,  $f(x) = 2 + \sin(2\sqrt{x})$

$$h = \frac{x-x_0}{n} = \frac{6-1}{10} = \frac{1}{2}$$

$$\begin{aligned} \int_a^b f(x)dx &= \int_1^6 (2 + \sin(2\sqrt{x})) dx \\ &= \frac{1}{2} \left[ f(1) + 2f\left(\frac{3}{2}\right) + 2f(2) + 2f\left(\frac{5}{2}\right) + 2f(3) + 2f\left(\frac{7}{2}\right) + 2f(4) + \right. \\ &\quad \left. + 2f\left(\frac{9}{2}\right) + 2f(5) + 2f\left(\frac{11}{2}\right) + f(6) \right] = 8.19385457 \end{aligned}$$

$$E_T = \left| \frac{-nh^3}{12} f''(\zeta) \right| \Rightarrow f''(\zeta) = \max(|f''(a)|, |f''(b)|)$$

$$f''(x) = \frac{d^2}{dx^2} (2 + \sin(2\sqrt{x})) = \frac{-\sin(2\sqrt{x}) \cos(2\sqrt{x})}{(\sqrt{x})^2} = \frac{-\sin(2\sqrt{x}) \cos(2\sqrt{x})}{2x(\sqrt{x})}$$

$$f''(1) = -0.701224 \quad , \quad f''(6) = 0.15746257$$

$$f''(\zeta) = \max(|-0.701224|, |0.15746257|) = 0.701224$$

$$\therefore E_T = \left| \frac{-10 \left(\frac{1}{2}\right)^3}{12} * 0.701224 \right|$$

$$\therefore E_T = 0.73014$$