Chapter five

Numerical Differentiation and Integration

B. Numerical Differentiation of Newton Backward Formula

$$f(x) = f(x_n) + (x - x_n)\frac{\nabla f_n}{1!\,h} + (x - x_n)(x - x_{n-1})\frac{\nabla^2 f_n}{2!\,h^2} + \cdots + (x - x_n)(x - x_{n-1})\ldots(x - x_1)\frac{\nabla^n f_n}{n!\,h^n} \ldots\ldots\ldots\ldots (6)$$

Example: find $f'(5.5)$, $f''(5.5)$, *and* $f'(6)$ *from the table :*

x	2	3	4	5	6
$f(x)$	5	10	17	26	37

Solution : we used Newton Backward Different formula (chapter four) to get the following table:

x	f	∇f_0	$\nabla^2 f_0$	$\nabla^3 f_0$	$\nabla^4 f_0$
2	5				
3	10	5	2	0	
4	17	7	2	0	0
5	26	9	2		
6	37	11			

$$q = \frac{x - x_n}{h} = \frac{5.5 - 6}{1} = -0.5$$

$$f'(x) = \frac{1}{h}\left[\nabla f_n + \frac{1}{2}(2q+1)\nabla^2 f_n + \frac{1}{6}(3q^2 + 6q + 2)\nabla^3 f_n + \frac{1}{12}(2q^3 + 9q^2 + 11q + 3)\nabla^4 f_n + \cdots\right]$$

$$f'(5.5) = \frac{1}{1}\left[11 + \frac{1}{2}\big(2(-0.5) + 1\big) * 2 + \frac{1}{6}(3(-0.5)^2 + 6(-0.5) + 2) * 0 + 0\right]$$

$$f'(5.5) = 11$$

$$f''(x) = \frac{1}{h^2}\left[\nabla^2 f_n + (q+1)\nabla^3 f_n + \frac{1}{12}(6q^2 + 18q + 11)\nabla^4 f_n + \cdots\right]$$

$$f''(5.5) = \frac{1}{(1)^2}[2 + 0 + 0] = 2$$

$$f'(x_n) = \frac{1}{h}\left[\nabla f_n - \frac{1}{2}\nabla^2 f_n + \frac{1}{3}\nabla^3 f_n + \frac{1}{4}\nabla^4 f_n + \cdots\right]$$

$$f'(6) = \frac{1}{1}\left[11 + \frac{1}{2} * 2 + \frac{1}{3} * 0 + \frac{1}{4} * 0\right] = 12$$