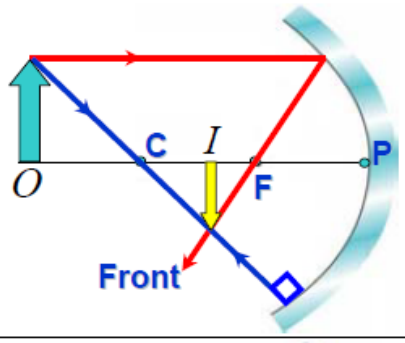
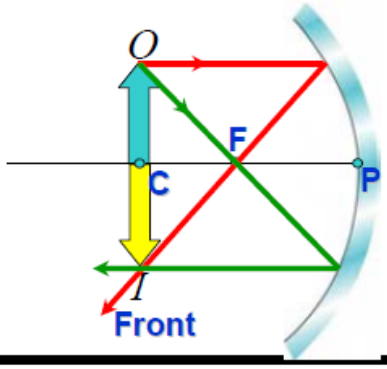
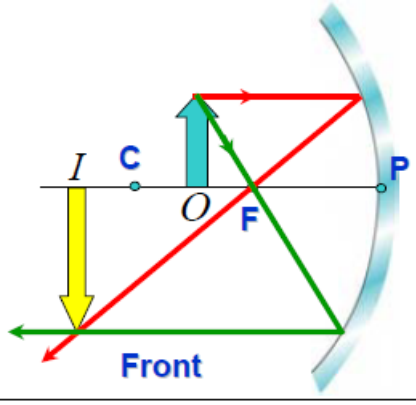
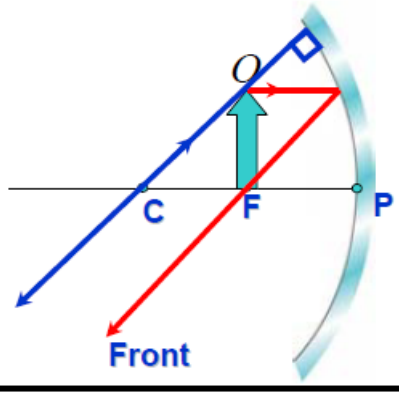
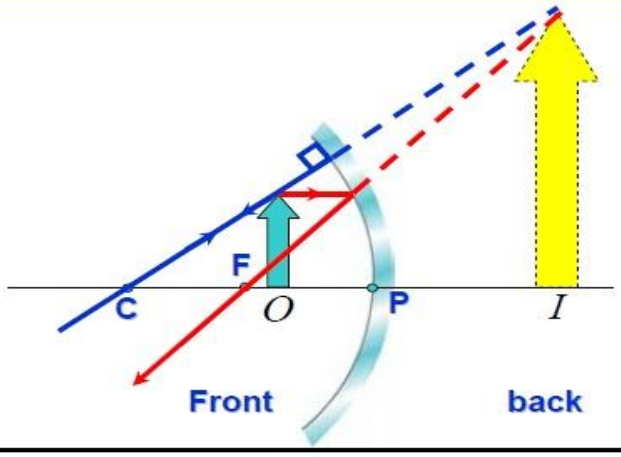
5.2. Images formed by a concave mirrors

Object distance, u	Ray diagram	Image property
$u > r$		○ Real ○ Inverted ○ Diminished ○ Formed between point C and F.
$u = r$		○ Real ○ Inverted ○ Same size ○ Formed at point C.

Object distance, u	Ray diagram	Image property
$f < u < r$		○ Real ○ Inverted ○ Magnified ○ Formed at a distance greater than CP.
$u = f$		○ Real ○ Formed at infinity.

Object distance, u	Ray diagram	Image property
$u < f$		○ Virtual ○ Upright ○ Magnified ○ Formed at the back of the mirror

○ Linear (lateral) magnification of the spherical mirror, M is defined as *the ratio between image height, h_i and object height, h_o*

$$M = \frac{h_i}{h_o} = -\frac{v}{u}$$

where
v : image distance from pole
u : object distance from pole

Negative sign indicates that the object and image are on opposite sides of the principal axis (refer to the real image), If h_o is positive, h_i is negative.

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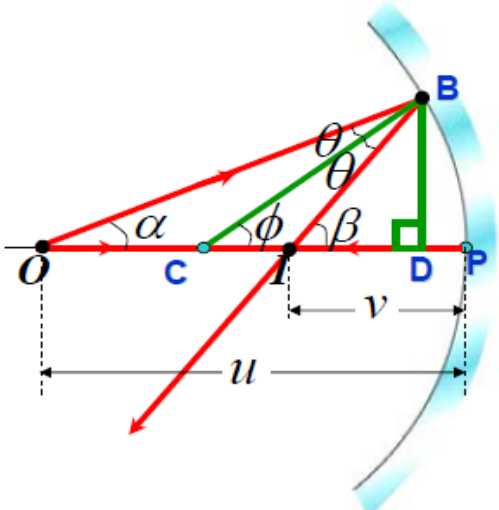
6.2. Derivation of Spherical Mirror Equation

○ Figure below shows an object O at a distance u and on the principal axis of a concave mirror. A ray from the object O is incident at a point B which is close to the pole P of the mirror.



○ From the figure,

△BOC $\Rightarrow \phi = \alpha + \theta$ ……(1)

△BCI $\Rightarrow \beta = \phi + \theta$ ……(2)

then, eq. (1)-(2) :

$$\phi - \beta = \alpha - \phi$$

$$\alpha + \beta = 2\phi \quad \text{……(3)}$$

By using △BOD, △BCD and △BID thus

$$\tan\alpha = \frac{BD}{OD}\,;\ \tan\phi = \frac{BD}{CD}\,;\ \tan\beta = \frac{BD}{ID}$$

○ By considering point B very close to the pole P, hence

$$\tan\alpha \approx \alpha\,;\ \tan\phi \approx \phi\,;\ \tan\beta \approx \beta$$

$$OD \approx OP = u\,;\ CD \approx CP = r\,;\ ID \approx IP = v$$

then

$$\alpha = \frac{BD}{u}\ ;\ \phi = \frac{BD}{r}\ ;\ \beta = \frac{BD}{v}$$

Substituting this value in eq. (3)

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therefore

$$\frac{BD}{u}+\frac{BD}{v}=2\left(\frac{BD}{r}\right)$$

$$\frac{1}{u}+\frac{1}{v}=\frac{2}{r} \quad \text{where} \quad r=2f$$

$$\boxed{\frac{1}{f}=\frac{1}{u}+\frac{1}{v}} \Rightarrow \textbf{Equation (formula) of spherical mirror}$$

- Table below shows the sign convention for equation of spherical mirror .

Physical Quantity	Positive sign (+)	Negative sign (-)
Object distance, u	Real object (in front of the mirror)	Virtual object (at the back of the mirror)
Image distance, v	Real image (same side of the object)	Virtual image (opposite side of the object)
Focal length, f	Concave mirror	Convex mirror
Linear magnification, M	Upright (erect) image	Inverted image

Example 3:

An object is placed 10 cm in front of a concave mirror whose focal length is 15 cm. Determine

a. the position of the image.

b. the linear magnification and state the properties of the image.

Solution: $u=+10\ cm, f=+15\ cm$

a. By applying the equation of spherical mirror, thus

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$$

$$\frac{1}{15}=\frac{1}{10}+\frac{1}{v}$$

$$v=-30\ cm$$

The image is 30 cm from the mirror on the opposite side of the object (or 30 cm at the back of the mirror).

b. The linear magnification is given by

$$M=-\frac{v}{u}=-\frac{(-30)}{10}$$

$$M=3$$

The properties of the image are

- ✓ **Virtual**
- ✓ **Upright**
- ✓ **Magnified**

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